4-ε Expansion

The critical exponents "stick" to their mean-field values above the upper critical dimension d_{uc}, but assume a nontrivial dimensionality dependence for $d < d_{uc}$. Most remarkably, their values prove to be smooth, analytical functions of the dimensionality d. This crucial observation, first made by Wilson and Fisher, led to the formulation of a systematic and controlled RG approach based on an expansion in $\varepsilon = d_{uc} - d$.

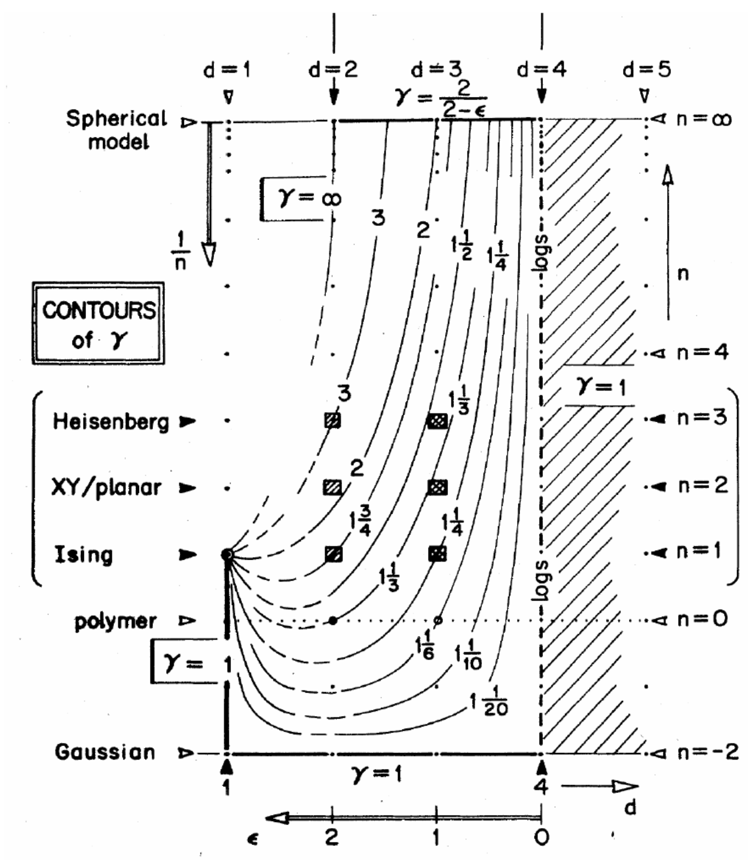

FIG. 3 Contours of constant susceptibility exponent γ in the (d, n) plane. The location of the $\gamma = \infty$ and other contours is uncertain in the region $d \simeq 2$, $n \geq 2$.

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We have already seen that the Gaussian theory provides an essentially exact formulation for the computation of critical exponents above the upper critical dimension ($d_{uc} = 4$ for the Ising model we consider here), since all higher order terms in the Landau action prove to be irrelevant operators. This is no longer true $d < d_{uc}$, where the situation becomes more complicated. Luckily, **most** terms still remain irrelevant (e.g. $k^4\phi^2$, or ϕ^n, with $n > 4$), but the $u\phi^4$ term now becomes relevant, since our power-counting analysis (based on the Gaussian model) already indicates that $u(b) = b^{4-d}u$ grows under rescaling.

The situation is now much more complicated. First, in contrast to the Gaussian model, the theory augmented by the $u\phi^4$ term cannot be solved in closed form. Second, the existence of an additional relevant operator now seems to bring into question the very validity of Kandanoff scaling. The solution of the problem is far from obvious, and this is the main reason why it took years from the original ideas of Widom and Kandanoff, until Wilson and Fisher realized that one can use the quantity $\varepsilon = d_{uc} - d$ as a small parameter to control the theory. Their idea was that, although physical systems live in integer dimensions, the mathematical problem of computing the partition function (i.e. critical exponents) can be analytically continued to arbitrary dimension d. This indeed proves to be the case, as one can see from the above figure (taken from the Michael Fisher's 1974 Rev. Mod. Phys. article), showing the smooth evolution of the susceptibility exponent γ for various models as a function of dimensionality d and the number of components of the order parameter n.

Perturbative RG in $d = 4 - \varepsilon$

We now proceed with implementing our RG program in $4 - \varepsilon$ dimensions, where the effects of the "interaction" u are small, and thus can be incorporated in a perturbative fashion. This does not mean that we are doing simple perturbation theory! What we do is the computation of the β-function in a perturbative fashion, i.e. in the regime where its form assumes a small deviation from that predicted by Gaussian theory.

As in the treatment in the Gaussian model, we break the order parameter field in its long wavelength and short wavelength components

$$\phi(\mathbf{k}) = \phi_{long}(\mathbf{k}) + \phi_{short}(\mathbf{k}),$$

with

$$\phi_{long}(\mathbf{k}) = \begin{cases} \phi(\mathbf{k}), & 0 < k < \Lambda/b \\ 0, & \Lambda/b < k < \Lambda \end{cases} ; \quad \phi_{short}(\mathbf{k}) = \begin{cases} 0, & 0 < k < \Lambda/b \\ \phi(\mathbf{k}), & \Lambda/b < k < \Lambda \end{cases},$$

and formally integrate over the short-wavelength components [We use this notation to avoid constantly having to write the limits of integration.]. In contrast to the Gaussian model, the Action is not any longer quadratic in ϕ_{short}, so the desired computation cannot be done exactly. Instead, we perform it perturbatively in the nonlinear u-term, as follows. In momentum space the Action takes the form (for simplicity, we consider $j = 0$)

$$S[\phi] = S_o[\phi_{long}] + S_o[\phi_{short}] + S_{int}[\phi_{long}, \phi_{short}],$$

with

$$\begin{aligned} S_o[\phi_{long}] &= \frac{1}{2}\int \frac{d\mathbf{k}}{(2\pi)^d}\phi_{long}(\mathbf{k})\,[r{+}k^2]\,\phi_{long}(\mathbf{k}); \\ S_o[\phi_{short}] &= \frac{1}{2}\int \frac{d\mathbf{k}}{(2\pi)^d}\phi_{short}(\mathbf{k})\,[r{+}k^2]\phi_{short}(\mathbf{k}), \end{aligned}$$

and

$$S_{int}[\phi_{long}, \phi_{short}] = \frac{u}{4}\int \frac{d\mathbf{k}_1}{(2\pi)^d}\frac{d\mathbf{k}_2}{(2\pi)^d}\frac{d\mathbf{k}_3}{(2\pi)^d}\frac{d\mathbf{k}_4}{(2\pi)^d}(2\pi)^d\delta(\mathbf{k}_1{+}\mathbf{k}_2{+}\mathbf{k}_3{+}\mathbf{k}_4)\phi(\mathbf{k}_1)\phi(\mathbf{k}_2)\phi(\mathbf{k}_3)\phi(\mathbf{k}_4).$$

Formally integrating out over ϕ_{short}, the partition function takes the form

$$Z = \int D\phi_{long}\exp\{-\widetilde{S}[\phi_{long}]\},$$

with

$$\begin{aligned} \widetilde{S}[\phi_{long}] &= -\ln\int D\phi_{short}\exp\{-S[\phi_{long}+\phi_{short}]\} \\ &= S_o[\phi_{long}] - \ln\langle\exp\{-S_{int}[\phi_{long}+\phi_{short}]\}\rangle_{S_o[\phi_{short}]} - \ln Z^o_{short}. \end{aligned}$$

We have already seen (when looking at the Gaussian model) that the constant

$$-\ln Z^o_{short} = -\ln\int D\phi_{short}\exp\{-S_o[\phi_{short}]\}$$

is a smooth function of parameters, and thus can be ignored in the critical regime. In these expression, the averages $\langle\cdots\rangle_{S_o[\phi_{short}]}$ are taken with respect to the Gaussian action $S_o[\phi_{short}]$ of short-wavelength fluctuations. These expressions are difficult to compute in general, but we again resort to perturbation theory (i.e. "cumulant expansion") in $S_{int} \sim u\phi^4$

$$\begin{aligned} \delta\widetilde{S}[\phi_{long}] &= -\ln\langle\exp\{-S_{int}[\phi_{long}+\phi_{short}]\}\rangle_{S_o[\phi_{short}]} \\ &\approx \langle S_{int}[\phi_{long}+\phi_{short}]\rangle_{S_o[\phi_{short}]} \\ &- \frac{1}{2}\left[\left\langle S^2_{int}[\phi_{long}+\phi_{short}]\right\rangle_{S_o[\phi_{short}]} - \langle S_{int}[\phi_{long}+\phi_{short}]\rangle^2_{S_o[\phi_{short}]}\right] + \cdots \end{aligned}$$

To linear order we find

$$\delta\widetilde{S}[\phi_{long}] \approx \frac{u}{4}\int \frac{d\mathbf{k}_1}{(2\pi)^d}\frac{d\mathbf{k}_2}{(2\pi)^d}\frac{d\mathbf{k}_3}{(2\pi)^d}\frac{d\mathbf{k}_4}{(2\pi)^d}(2\pi)^d$$
$$\times\, \delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)\, \langle\phi(\mathbf{k}_1)\phi(\mathbf{k}_2)\phi(\mathbf{k}_3)\phi(\mathbf{k}_3)\rangle_{S_o[\phi_{short}]}\,.$$

However, since ϕ is now a sum of two terms ϕ_{long} and ϕ_{short}, we get contributions to $\delta\widetilde{S}[\phi_{long}]$ with different powers of ϕ_{long}. Clearly, the constant term (power $(\phi_{long})^0 \sim const.$) again provides a contribution to the smooth background part of the free energy, and can be safely ignored. The one with ϕ^4_{long} is simply the bare interaction vertex for ϕ_{long}. The only nontrivial contribution to this order is the "mass" renormalization corresponding to the "Hartree" diagram corresponding to the ϕ^2_{long}. More explicitly, this contribution takes the form

$$\delta\widetilde{S}^{(2)}[\phi_{long}] = \frac{u}{4}6\int \frac{d\mathbf{k}_1}{(2\pi)^d}\frac{d\mathbf{k}_2}{(2\pi)^d}\frac{d\mathbf{k}_3}{(2\pi)^d}\frac{d\mathbf{k}_4}{(2\pi)^d}(2\pi)^d$$
$$\times\, \delta(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)\phi_{long}(\mathbf{k}_1)\phi_{long}(\mathbf{k}_2)\, \langle\phi_{short}(\mathbf{k}_3)\phi_{short}(\mathbf{k}_4)\rangle_{S_o[\phi_{short}]}\,.$$

Using the fact that

$$\langle\phi_{short}(\mathbf{k}_3)\phi_{short}(\mathbf{k}_4)\rangle_{S_o[\phi_{short}]} = (2\pi)^d\delta(\mathbf{k}_3+\mathbf{k}_4)G_o(\mathbf{k}_3),$$

and $\phi_{long}(\mathbf{k}) = \phi_{long}(-\mathbf{k})$ (since $\phi_{long}(\mathbf{x})$ is real), we finally obtain

$$\delta\widetilde{S}^{(2)}[\phi_{long}] = \frac{1}{2}3u\int_o^{\Lambda/b}\frac{d\mathbf{k}}{(2\pi)^d}\phi^2_{long}(\mathbf{k})\int_{\Lambda/b}^{\Lambda}\frac{d\mathbf{q}}{(2\pi)^d}\frac{1}{r+q^2}\,.$$

Finally, after rescaling the momenta $\Lambda/b \longrightarrow \Lambda$ in the $\mathbf{k}$-integration (just as we have done in the analysis of the Gaussian model), we find

$$r(b) = b^2\left[r + 3u\int_{\Lambda/b}^{\Lambda}\frac{d\mathbf{q}}{(2\pi)^d}\frac{1}{r+q^2} + O(u^2)\right].$$

To calculate the β-function, consider an infinitesimal momentum shell integration $b = e^{\delta\ell}$, with $\delta\ell \ll 1$, so $b \approx 1+\delta\ell$, and the integral can be approximated as

$$\int_{\Lambda/b}^{\Lambda}\frac{d\mathbf{q}}{(2\pi)^d}\frac{1}{r+q^2} \approx \frac{K_d\Lambda^d}{r+\Lambda^2}\delta\ell + O(\delta\ell^2).$$

We can write

$$\begin{aligned} r(\ell+\delta\ell) &\approx (1+2\delta\ell)\left[r(\ell)+\frac{3uK_d\Lambda^d}{r+\Lambda^2}\delta\ell\right]+O(u,\delta\ell^2) \\ &\approx r(\ell)+\left[2r(\ell)+\frac{3uK_d\Lambda^d}{r+\Lambda^2}\right]\delta\ell+O(u,\delta\ell^2). \end{aligned}$$

In the continuum limit $\delta\ell \longrightarrow 0$ we find

$$\beta_r = \frac{dr}{d\ell} = 2r + 3\Omega_d \frac{u}{r+\Lambda^2} + O(u^2).$$

where $\Omega_d = K_d\Lambda^d$.

To get a leading-order renormalization for the interaction vertex we need to carry out the cumulant expansion to $O(u^2)$. We again use the Wick's theorem, and the result (left as a homework problem) is

$$\beta_u = \frac{du}{d\ell} = (4-d)u - 9\Omega_d \frac{u^2}{(r+\Lambda^2)^2} + O(u^2).$$

Gaussian fixed point

We are now in a position to analyze our RG flows. We know that the critical point is identified as an unstable **fixed point** of the RG flows. What are the fixed point in the present case? To answer this question, we look for the solution of the equations

$$\begin{aligned} 0 &= 2r + 3\Omega_d \frac{u}{r+\Lambda^2}; \\ 0 &= (4-d)u - 9\Omega_d \frac{u^2}{(r+\Lambda^2)^2}. \end{aligned}$$

As in the Gaussian model, there is a solution $u^* = r^* = 0$. We call this the Gaussian fixed point. Let us have a look at the form of the RG flows around this fixed point. How to do this? The considered RG equations are nonlinear differential equations, and their solution describes the family of RG flows. At first glance it seems difficult to analytically determine the form of these flows. But in the vicinity of a fixed point the problem is simpler, as one can linearize the equations, in which case they reduce to a system of first order linear differential equations which can be easily.

Let us linearize the equations around the Gaussian fixed point

$$\frac{dr}{d\ell} \approx 2r + au;$$
$$\frac{du}{d\ell} = (4-d)u,$$

where $a = 3\Omega_d/\Lambda^2$. It is convenient to write these equations in matrix form

$$\frac{d\mathbf{z}(\ell)}{d\ell} = \mathbf{M}_o \mathbf{z}(\ell),$$

where

$$\mathbf{z}(\ell) = \begin{bmatrix} r(\ell) \\ u(\ell) \end{bmatrix} \text{ and } \mathbf{M}_o = \begin{bmatrix} 2 & a \\ 0 & 4-d \end{bmatrix}.$$

Such a coupled set of linear equations can in general be solved by eigenvalue analysis, and we look for eigenvalues $\lambda^o_{1/2}$ of the (Gaussian) matrix which are found as a solution of the eigenvalue equation

$$\begin{vmatrix} 2-\lambda^o_1 & a \\ 0 & 4-d-\lambda^0_2 \end{vmatrix} = 0,$$

or (note that the eigenvalues do not depend on the constant $a = 3\Omega_d/\Lambda^2$)

$$(2-\lambda^o_1)\,(4-d-\lambda^o_2) = 0.$$

giving

$$\lambda^o_1 = 2 > 0$$
$$\lambda^o_2 = 4-d.$$

Just as we concluded from the analysis of the Gaussian model, the second eigenvalue becomes positive (relevant) for $d < 4$. The corresponding eigenvectors are

$$\mathbf{e}^o_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; \ \ \mathbf{e}^o_2 = \begin{bmatrix} -a \\ d-2 \end{bmatrix}.$$

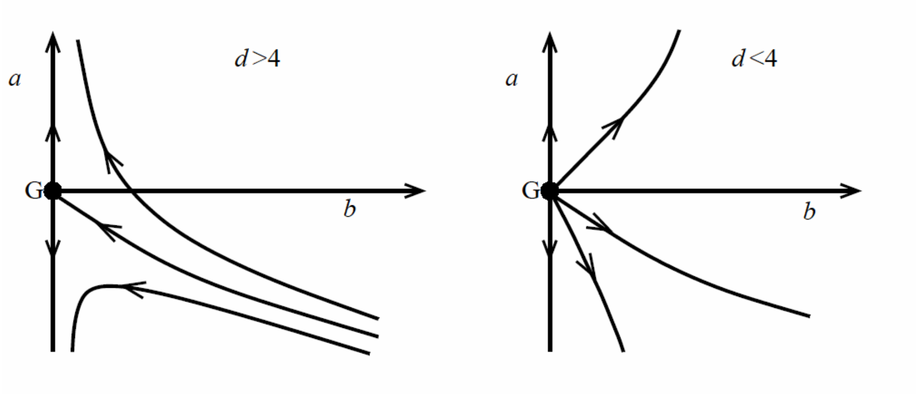
Any vector $\mathbf{z}(\ell)$ can be expanded in the eigenvalue basis

$$\mathbf{z}(\ell) = z_1(\ell)\mathbf{e}^o_1 + z_2(\ell)\mathbf{e}^o_2.$$

Plugging this in the differential equation, we find

$$z_{1/2}(\ell) = e^{\lambda^o_1 \ell}.$$

The resulting structure of flows is shown in the figure.

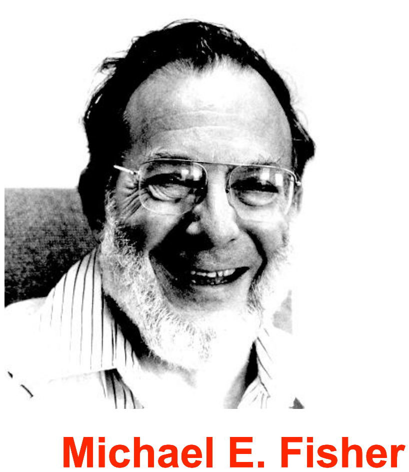


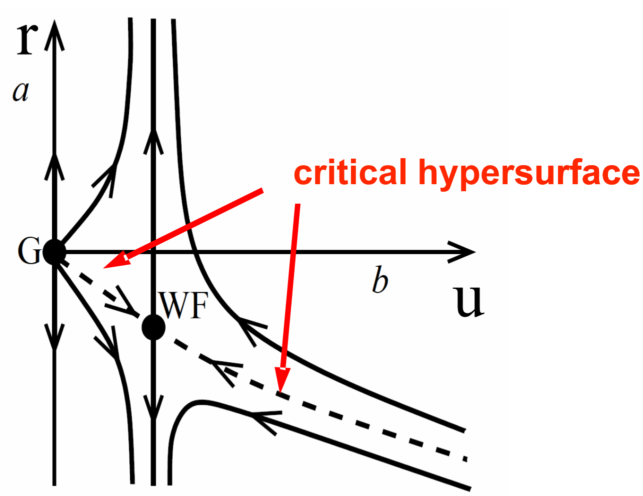
Wilson-Fisher fixed point

The situation seems discouraging, as we find two relevant eigenvalues. At first glance, the scaling hypothesis of Kadanoff does not work. What is wrong? The answer was provided by the ground-breaking observation of Wilson and Fisher, who noticed that below four dimensions **another** fixed point emerges, with all the right properties. Expecting that for $\varepsilon = 4 - d$ small this second fixed point is close to the Gaussian one, we expand the RG equations to quadratic order

$$\frac{dr}{d\ell} \approx 2r + au - bur;$$
$$\frac{du}{d\ell} \approx \varepsilon u - 3bu^2,$$

where $\varepsilon = 4 - d$, and $b = 3\Omega_d/\Lambda^4$. Note that to this order the second equation does not





depend on r, and we can immediately find a new (Wilson-Fisher) fixed point at

$$u^* = \varepsilon/3b \text{ and } r^* = -au^*/2 = -\varepsilon a/6b.$$

Next, expand the β-function around the nontrivial fixed point defining $\delta r = r - r^*$ and $\delta u^* = u - u^*$

$$\frac{d}{d\ell}\delta r = (2 - bu^*)\delta r + (a - br^*)\delta u;$$
$$\frac{d}{d\ell}\delta u = (\varepsilon - 6bu^*)\delta u.$$

The matrix to be diagonalized now is

$$\mathbf{M} = \begin{bmatrix} 2 - bu^* & a - br^* \\ 0 & \varepsilon - 6bu^* \end{bmatrix},$$

and we immediately find

$$\lambda_1 = 2 - bu^* = 2 - \varepsilon/3 > 0;$$
$$\lambda_2 = \varepsilon - 6bu^* = -\varepsilon < 0!!!!$$

The flows corresponding to the Wilson-Fisher (WF) and the Gaussian (G) fixed point are shown in the figure.

Field scaling and exponents

An external symmetry breaking field is generally a relevant perturbation which vanishes at the critical point. It can therefore be treated as a small perturbation. For a uniform external field the corresponding contribution to the Landau action takes the form

$$S_j = -j \int d\mathbf{x}\, \phi(\mathbf{x}) = -j\phi(\mathbf{k} = 0).$$

Since it only couples to the $\mathbf{k} = 0$ component of the order parameter, it is not directly affected by momentum shell integration, which eliminates high momentum components of the order parameter field. The only renormalizations then arise from the length scale and field renormalization, precisely as we described in the Gaussian model, and we find

$$\frac{dj}{d\ell} = (\frac{d}{2} + 1)j,$$

i.e.

$$\lambda_j = \frac{d}{2} + 1.$$

Just as the standard Kadanoff scheme, we are therefore left with only two relevant eigenvalues $\lambda_t = \lambda_1$and λ_j, both of which we have computed within the ε-expansion. We can immediately use the Kadanoff scaling relations to compute the values of all the critical exponents, and we find

$$\nu = \frac{1}{2} + \frac{\varepsilon}{12}; \quad \alpha = \frac{\varepsilon}{6}; \quad \gamma = 1 + \frac{\varepsilon}{6}; \quad \beta = \frac{1}{2} - \frac{\varepsilon}{6}; \quad \delta = 3 + \varepsilon.$$

These results provide surprisingly good estimates if we extrapolate them to $d = 3$ (i.e. $\varepsilon = 1$).

Universality restored

We pause to fully appreciate the incredible beauty and significance of these landmark result. The first important observation we make is that, in contrast to the Gaussian fixed point, the WF fixed point has *only one* relevant eigenvalue. It corresponds to the combination of the r and u terms in the Action which represent the only *relevant* operator. The other eigenvalue is negative, thus corresponding to an *irrelevant* operator. This can be more fully appreciated if we have a look at the structure of the RG flows, as shown in the figure. There can be clearly seen a line in the r-u plane corresponding to the irrelevant eigenvalue and which, as we will not show, corresponds to the *critical hypersurface* (dashed line in figure).

The flows divide in three distinct groups, as follows:

1. If the "bare" values of r and u are "above" the critical hypersurface, then $r \longrightarrow +\infty$ and $u \longrightarrow u^*$. This corresponds to the high temperature (paramagnetic) phase of the system.

2. If the "bare" values of r and u are "below" the critical hypersurface, then $r \longrightarrow -\infty$ and $u \longrightarrow u^*$. This corresponds to the low temperature (ordered) phase of the system.

3. If the "bare" values of r and u are anywhere *on the critical hypersurface*, then $r \longrightarrow r^*$ and $u \longrightarrow u^*$. This corresponds to critical point itself. Note that there is an entire

manifold of bare values for r and u that corresponds to the critical point of the system. These correspond to different microscopic models which all find themselves precisely at the critical temperature.

Crossover phenomena and the Ginzburg region in RG language

We are now equipped to discuss how the system behaves as the transition is approached. We anticipate that far enough from the transition (in fact everywhere except in a very narrow critical region), the behavior is well represented by Landau theory, as fluctuation corrections are small. How does this translate in the RG flow language? To answer this question, imagine the system is very close to the critical temperature. In RG language this means that the initial condition (bare values for r and u) are very close to the critical hypersurface. Upon rescaling, the system flows "towards" the WF fixed point for a very long time, until it eventually decides to "peel off" and flow towards $r \longrightarrow +\infty$. But by this time the "memory" of initial conditions (bare coupling constants) is completely forgotten! This is precisely the manifestation of universality Kadanoff has correctly anticipated.

On the other hand, if the system is farther away from the critical temperature, then the initial conditions in the r-u plane are not too close to the critical hypersurface. Imagine that we start with large values of r and u. Since the Gaussian and the Wilson-Fisher fixed points are very close (at least for $\varepsilon = d - 4 \ll 1$), the coupling constant u appears to flow to zero for a very long time, until it finally saturates to $u = u^*$. Until the flows reach the immediate vicinity of the WF fixed point, they assume exactly the same form as in the Gaussian model. Indeed, if we are far enough from the transition, then the system cannot "distinguish" or "resolve" the Gaussian from the Wilson-Fisher fixed point, and the behavior is extremely well described by the Gaussian theory. In this regime, therefore, all predictions of Landau theory apply, and fluctuation corrections are small.

Generalizations: $O(n)$ theory

The procedure can be easily generalized to many more complicated situations. For example, for isotropic magnets the order parameter (local magnetization) is a three component vector field that can smoothly rotate in any direction. This is commonly called the "Heisenberg" model. The appropriate generalization to an n-component field is the so-called $O(n)$ φ^4-theory with Landau action

$$S[\phi]=\frac{1}{2}\sum_{\alpha=1}^{n}\int d\mathbf{x}\,\phi_\alpha(\mathbf{x})\,[r-\nabla^2]\,\phi_\alpha(\mathbf{x})+\frac{u}{4}\int d\mathbf{x}\,\left(\sum_{\alpha=1}^{n}\phi_\alpha^2(\mathbf{x})\right)^2-\sum_{\alpha=1}^{n}\int d\mathbf{x}\,j_a(\mathbf{x})\phi_\alpha(\mathbf{x}).$$

The lowest order ε-expansion can easily be worked out for this model (homework problem) giving

$$\frac{dr}{d\ell}=\beta_r^{(n)}=2r+(n+2)\Omega_d\frac{u}{r+\Lambda^2};$$
$$\frac{du}{d\ell}=\beta_u^{(n)}=(4-d)u-(n+8)\Omega_d\frac{u^2}{(r+\Lambda^2)^2}.$$

The eigenvalues now take the form

$$\lambda_1=2-\frac{n+2}{n+8}\varepsilon+O(\varepsilon^2);\quad \lambda_2=-\varepsilon+O(\varepsilon^2).$$

Perturbations and crossovers

In real materials, there are often perturbations that weakly violate some symmetry of the Hamiltonian. For example, many magnets are almost completely isotropic, except for a small "cubic" anisotropy resulting from crystal fields, which favors certain crystallographic directions for magnetization. How do these affect the critical behavior? We have seen that within Landau theory the critical exponents are the same for all symmetry classes, for example for Heisenberg (isotropic) and Ising (easy axis) magnets. When fluctuation corrections are incorporated within our RG approach, we have seen that critical exponents do depend on the number of components n.

But what happens if a small symmetry breaking field is added? How do we describe this effect within our RG scheme? Consider, for example "cubic anisotropy", which can result from crystal fields, and which takes the form

$$\delta S_v=\frac{v}{4}\int d\mathbf{x}\,\sum_{\alpha=1}^{n}\phi_\alpha^4(\mathbf{x}).$$

Note that such a perturbation does not respect the rotational symmetry of the original model. Its effects can be analyzed by examining how the coupling constant v renormalizes under rescaling, and using the same ε-expansion approach we have used before. The results show that for a sufficiently small number of components ($n < 4$), the isotropic ($v = 0$) fixed point is stable, and the perturbation is an irrelevant operator. Therfore such a term does not affect the behavior of Heisenberg ($n = 3$) or even XY ($n = 2$) magnets.

Another example is the single ion anisotropy, where the perurbation takes the form

$$\delta S_g = -\frac{1}{2} g \int d\mathbf{x}\ \phi_1^2(\mathbf{x}).$$

Such a term will be present, for example, when one particular direction (call it $\alpha = 1$) is energetically more (or less for $g < 0$) favorable for ordering. We can again apply the ε-expansion approach. This calculation is similar as what we have done before, and is left for a homework problem. The net conclusion, however, is that such a symmetry breaking term is *always a relevant perturbation*, and the isotropic fixed point is generically unstable. What happens when g is very small? New fixed points emerge, reflecting behavior of models with lower symmetry. For $g > 0$ the spins preperentially order in the direction $\alpha = 1$, and one get critical behavior identical to that of the Ising ($n = 1$) model. In the opposite limit ($g < 0$) the spins poult like to order in the "plane" perpendicular to $\alpha = 1$ ("XY plane"), and one gets critical behavior of the $n - 1$ component model.

If the bare value of the symmetry breaking perturbation is weak, then we find a prototypical example of a *crossover phenomenon*. If one considers the system not too close to the critical temperature (e.g. critical hypersurface), then the form of the flows is almost identical as for the isotropic system. We say that in a broad range of temperatures the behavior of the system is controlled by an unstable fixed point. Only if the system is very close to the critical temperature we see the effects of the symmetry breaking perturbation. Within the critical manifold the system then "crosses-over" from an unstable to a stable fixed point. To gauge the relative importance of different symmetry breaking perturbations one, therefore, has to classify all possible fixed points within the critical hypersurface manifold, and examine their relative stability. A nice discussion of crossovers corresponding to different symmetry classes can be found in the 1974 review article by Michael Fisher.