Homework Assignment #1.

Problem 1. Use thermodynamic arguments (see book by Fermi) to show that

$$C_p = \left(\frac{\partial Q}{\partial T}\right)_p = \left(\frac{\partial E}{\partial T}\right)_p + p\left(\frac{\partial V}{\partial T}\right)_p.$$

Problem 2. Use thermodynamic arguments to show (e.g. see book by Fermi or Chaikin and Lubensky) that in equilibrium the following stability conditions hold

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_p \geq 0;\ \kappa_T = -\left(\frac{\partial V}{\partial P}\right)_T \geq 0;\ \kappa_S = -\left(\frac{\partial V}{\partial P}\right)_S \geq 0.$$

Problem 3. Consider a system consisting of two Ising spins $S_i = \pm 1$**;** $i = 1, 2$, interacting through a ferromagnetic interaction $J > 0$, as described by a Hamiltonian

$$H = -JS_1S_2 - h(S_1 + S_2).$$

Here, h is an external magnetic field coupling to the spins.

(a) Calculate the partition function of the system as a function of T and h, and from it obtain the free energy ($k_B = 1$)

$$F(T, h) = -T \ln Z(T, h).$$

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(b) The calculate the magnetization per spin

$$m(T, h) = \frac{1}{2}\frac{\partial F}{\partial h},$$

and the spin susceptibility

$$\chi(T) = \left(\frac{\partial m}{\partial h}\right)_{h=0}.$$

(c) Show that to leading order at high temperatures

$$\chi^{-1}(T) \approx T - \Theta,$$

where the "Curie constant" $\Theta = J$. Plotting the inverse spin susceptibility to reveal such "Curie tails" is often used in experiments to identify the presence of local magnetic moments in the system. The value of the Curie constant Θ is used to estimate the spin-spin interaction.

(d) Use the full expression for $\chi(T)$ to plot the inverse susceptibility $\chi^{-1}(T)$ as a function of temperature and show that for the considered system it remains finite and any finite temperature.

(e) Plot the magnetization as a function of temperature by choosing a small but nonzero value of h. What is the temperature below which the magnetization suddenly becomes larger? Why?

(d) Compare the results for $\chi(T)$ and $m(h, T)$ to those corresponding to $J = 0$, and discuss the difference and its physical origin. How do you expect the results to change if we considered $N > 2$ interacting spins? What do you expect to happen in the limit $N \longrightarrow \infty$?

Problem 4. Use the Van der Waals equation to calculate the critical behavior of the specific heat C_V, the isothermal compressibility κ_T, and the reduced density jump $\delta v = |V - V_c|/V_c$ (across the first order transition line). Determine the corresponding critical exponents.