## Homework Assignment \#6 - Solution

## Section 2

We are trying to fit the data of conductivity as function of temperature of the form:

$$
\begin{equation*}
\sigma(T)=\sigma_{0}(n, T=0)+m T^{\alpha} . \tag{1}
\end{equation*}
$$

With the use of $\alpha=2$, the dependence of conductivity on $T^{\alpha}$ is linear, i. e. we get straight lines as plotted in the following figure:


## Section 3

The fitting results for the previous section also gives us the following table for the conductivity $\sigma_{0}$ extrapolated to $T=0$ corresponding to different densities $n$ used:

| $n$ | 0.7 | 0.8 | 0.9 | 1.0 | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\sigma_{0}$ | $-5.38 \mathrm{e}-4$ | $5.94 \mathrm{e}-4$ | 0.00352 | 0.00787 | 0.0141 | 0.0235 | 0.0356 | 0.0472 | 0.0672 | 0.0908 |
| $n$ | 1.7 | 1.8 | 1.9 | 2.0 | 2.1 | 2.2 | 2.3 | 2.4 | 2.5 | 2.6 |
| $\sigma_{0}$ | 0.123 | 0.159 | 0.208 | 0.258 | 0.322 | 0.394 | 0.467 | 0.559 | 0.646 | 0.754 |
| $n$ | 2.7 | 2.8 | 2.9 | 3.0 |  |  |  |  |  |  |
| $\sigma_{0}$ | 0.847 | 0.961 | 1.09 | 1.22 |  |  |  |  |  |  |

As we expect that:

$$
\begin{equation*}
\sigma_{0}=a\left(n=n_{c}\right)^{\mu}, \tag{2}
\end{equation*}
$$

one can find $a=0.147, n_{c}=0.747, \mu=2.61$. The figure showing this fitting is presented as follow:


## Section 4

As we found $n_{c}=0.747 \times 10^{11} \mathrm{~cm}^{-2}$, we will focus on the data with densities around this value. The following figure shows that the curves $\sigma(T)$ on the $\log$-log plot will be straight at the density $n_{c}$ located somewhere between two values $n=0.65 \times 10^{11} \mathrm{~cm}^{-2}$ and $n=0.7 \times 10^{11} \mathrm{~cm}^{-2}$.


This value of $n_{c}$ may be considered to agrees with the value $n_{c}=0.747 \times 10^{11} \mathrm{~cm}^{-2}$ found in the section (3). In order to continue our work for the next sections, let's choose $n_{c}=0.65 \times 10^{11} \mathrm{~cm}^{-2}$. At this density, we can extract $x=2.39$.

## Section 5

In order to plot $\sigma(T) / \sigma_{c}(T) \sim \sigma(T) / T^{x}$ on a log-log scale, we choose $n_{c}=0.65 \times 10^{11} \mathrm{~cm}^{-2}$. At this point, as described previously, we got $x=2.39$. The figure shows that the curve corresponding to the critical density $n_{c}=0.65 \times 10^{11} \mathrm{~cm}^{-2}$ is horizontal. All the curves with $n<n_{c}$ curve down at $T \rightarrow 0$ corresponding to the insulating phase, while all the curves with $n>n_{c}$ curve up at $T \rightarrow 0$
indicating the metallic phase.


## Section 6

Using the data obtained for the section 5 and the assumption that:

$$
\begin{equation*}
T^{*}(n) \sim\left(\frac{\left|n-n_{c}\right|}{n_{c}}\right)^{\nu z}=\delta n^{\nu z}, \tag{3}
\end{equation*}
$$

we can find the values of $n \nu$, at which all the conductivities collapse onto a scaling function $f$, i.e.:

$$
\begin{equation*}
\sigma(n, T)=\sigma_{c}(T) f\left(\frac{T}{\delta n^{\nu z}}\right) . \tag{4}
\end{equation*}
$$

The following figure indicates explicitly the collapse mentioned above.


## Section 7

The $T^{*}(n)$ is plotted in the $n-T$ phase diagram as follow:


## Section 8

With the assumption (3), in the section 6 the exponent $\nu z=1.4$ for metallic regime, and $\nu z=1.3$ for insulating regime.

## Section 9

In the section 4 , we found $x=2.39$ while in the section 6 and 8 we found $\nu z=1.4$. Therefore, t the conductivity exponent is

$$
\begin{equation*}
\mu=\nu z x=3.346 . \tag{5}
\end{equation*}
$$

This value may be considered to agree with $\mu=2.61$ estimated in the section 3 .

