Homework Assignment #6.

Scaling Analysys of a Quantum Critical Point

In this assignment, we will borrow experimental data obtained by Dr. Dragana Popović (NHMFL/FSU), who performed transport measurements on silicon MOSFETs, as a function of carrier concentration n*, and temperature* T*. Although the experiment is done at finite temperature, a careful scaling analysis reveals the existence of a* $T = 0$ *quantum critical point, which in this case corresponds to a metal-insulator transition. Following Dr. Popović, we will carry out a such a scaling analysis, and extract the critical exponents characterizing the critical point. This approach, similar to that used by Ben Widom in the heroic days of thermal critical phenomena, is based on the phenomenological scaling hypothesis. It is seen to work remarkably well, despite the current lack of accepted microscopic theory for this phenomenon.*

1. The experimental data are available in Excel format on our Web page. The data are organized in columns, where each column represents the conductivity σ (in units of $\hbar/e^2$) for a given density n (in units of $10^{11} cm^{-2}$), as a function of temperature T (in degrees Kelvin). Download these data, then use whatever software you prefer to analyze it, using the following procedure.

2. Let us first examine the form of the conductivity in the metallic phase (high density regime). Here, we expect the temperature dependence to assume the following form

$$\sigma(T) = \sigma_o + mT^{\alpha}.$$

Plot these data as a function of T^{α}. Try different exponents α until you get a straight line. What is the value of α you obtain? The intercept obtained from such a plot is the conductivityσ_o *extrapolated* to $T = 0$.

3. Repeat the procedure for each density to obtain $\sigma_o(n)$. We expect

$$\sigma_o = a(n - n_c)^{\mu},$$

at some critical concentration $n = n_c$. Try to estimate n_c and the conductivity exponent μ by fitting $\sigma_o(n)$ to this form [many software programs offer a nonlinear fitting routine; you should use n_c, a, and μ as fitting parameters.

4. Next, we perform the full "dynamical scaling" analysis, as follows. First, plot a series of curves on a log-log plot, showing $\sigma(T)$ as a function of temperature T, for each different density n. We expect the conductivity to be a simple power-law function of temperature

$$\sigma_c(T) \sim T^x$$

only at the critical density. On a log-log plot, thus only the curve corresponding to $n = n_c$ will look like a straight line. This will allow you to immediately determine the critical concentration n_c. The slope is the exponent x. What do you get for x and n_c? Does it agree with the result of (3)?

5. Second, plot (again on a log-log scale) $\sigma(T)/\sigma_c(T) \sim \sigma(T)/T^x$ as a function of T. Now the curve corresponding to the critical density will be horizontal (the "separatrix"). The metallic curves will curve "up" at $T \longrightarrow 0$, while the insulating ones will curve "down" at $T \longrightarrow 0$.

6. Third, we want to collapse all the curves on a scaling function. More precisely, we want to collapse separately all the metallic and all the insulating curves. To do this, pick a density not too close to the transition. Then, plot the other curves as a function of $T/T^*(n)$. You have to determine a different $T^*(n)$ for each density until all the curves collapse onto a scaling function. Plot all the results in the scaling form; the scaling function consists of a metallic and an insulating "branch". We expect scaling to break down if we are too far from the quantum critical point, which is located at $n = n_c$ and $T = 0$. How broad is the region of densities where scaling seems to work?

7. The density-dependent temperature scale $T^*(n)$ represents a crossover temperature, above which we have the quantum critical region, and below which the system is in the metallic (for $n > n_c$) or the insulating (for $n < n_c$) regime. Plot $T^*(n)$ on the $n - T$ phase diagram. Label the location of the critical point, the metallic, insulating, and the quantum critical regime.

8. We expect the crossover temperature to behave as

$$T^*(n) \sim \delta n^{\nu z},$$

where the reduced density $\delta n = (n - n_c)/n_c$. To determine the exponent νz, plot $T^*(n)$ on a log-log scale as a function if $|\delta n|$. Do that separately for the metallic ($\delta n > 0$) and the insulating ($\delta n < 0$) regimes. The slope is then the exponent νz. What do you get?

9. From standard scaling considerations (see lecture notes), we expect the $T = 0$ conductivity exponent to be

$$\mu = \nu z\ x.$$

Does the value you obtain from this approach agree with your estimate of μ in part (3)?