# Homework Assignment #8.

**Problem 8.1.**

(a) Prove the validity of the Hubbard-Stratonovich (Gaussian) transformation

$$\exp\left\{\frac{1}{2}\sum_{ij} S_i K_{ij} S_j\right\} = \int \prod_{i=1}^{N} d\phi_i \exp\left\{-\frac{1}{2}\sum_{ij} \phi_i K_{ij} \phi_j + \sum_{ij} S_i K_{ij} \phi_i\right\},$$

whenever $K_{ij}$ is a real and symmetric matrix. To do this, note that in this case the matrix $K_{ij}$ can be diagonalized by a unitary transformation, and the integral on the right-hand side can be factored into a product of $N$ Gaussian integrals.

(b) Starting with the action $S[\phi]$ obtained through through the Hubbard-Stratonovich transformation

$$S[\phi] = \frac{1}{2}\sum_{ij} \phi_i K_{ij} \phi_j - \sum_i \ln\cosh\left(\sum_j K_{ij}\phi_j + h_i\right),$$

perform the momentum expansion to reduce the action to the Landau form

$$S[\phi] = \frac{1}{2}\int d\mathbf{x}\, \phi(\mathbf{x})\left[r - \nabla^2\right]\phi(\mathbf{x}) + \frac{u}{4}\int d\mathbf{x}\ \phi^4(\mathbf{x}) - \int d\mathbf{x}\, h(\mathbf{x})\phi(\mathbf{x}).$$

Dtermine the values of the Landau parameters $r$ and $u$ in terms of $K = \beta J$.

**Problem 8.2.**

(a) Consider an operator of the form

$$S_m = w_m \int d\mathbf{x}\ \phi^m(\mathbf{x}),$$

and carry out the power-counting analysis within a Gaussian model, to demonstrate that the corresponding scaling exponent is

$$\lambda_w(m) = m - (m-2)d/2.$$

(b) Carry out the analysis of dangerously irrelevant variables to show that the exponent $\delta = 3$ for all $d > d_{uc}$. [Hint: to determine $\delta$, concentrate to the behavior at the critical point ($r = 0$), and choose $b^{\lambda_j} j = 1$. The rest of the analysis is similar as for the calculation of $\beta$.]