PHY6937-02: Mid-Term Exam

Problem 1. Describe the behavior of the spin susceptibility as a function of temperature, for the following models (describe in words, you can also use a sketch to show the qualitative behavior):

5 (a) $d = 3$ Ising ferromagnet.

It diverges at the critical temperature as $\chi \sim (T - T_c)^{-\gamma}$. The exponent $\nu \approx 1.24$ is a nontrivial irrational number in $d = 3$.

5 (b) $d = 5$ Ising ferromagnet; how different is it from (a)? Why?

It diverges at the critical temperature as $\chi \sim (T - T_c)^{-\gamma}$. The exponent $\nu = 1$, assumes the exact mean-field value, becuase $d = 5$ is above the upper critical dimension $d_{uc} = 4$, for the Ising model.

5 (c) $d = 3$ Ising antiferromagnet.

The susceptibility does not diverge, but it displays a cusp for an antiferromagnet.

5 (d) $d = 1$ Ising ferromagnet.

There is no finite temperature phase transition for the $d = 1$ Ising model, and the susceptibility diverges (exponentially) at $T = 0$.

10 (e) Ferromagnetic Ising model on a cubic cluster of size (5x5x5). What is the difference from the behavior of bulk material, discussed in (a)-(d)? How does the behavior depend on the cluster size? Why?

There cannot be spontaneous symmetry breaking for a finite system. As the cluster size grows, the "equilibration time" grows exponentially with the system suze. Such "ergodicity breaking" is a neccessary condition for spontaneous symmetry breaking, and as such it exists only in the thermodynamic limit.

Problem 2. Write down the Landau-Ginzburg theory (equation os state) for a ferromagnet, and from it determine:

10 (a) The field-dependence of the magnetization at the critical temperature $T = T_c$.

10 (b) Temperature dependence of the spin susceptibility at high temperatures.

10 (c) The coercive field (spinodal line), as a function of $r \sim (T_c - T)$.

These questions have already been discussed in the lecture notes, and as part of the homework problems.

20 **Problem 3.** Present the Widom scaling argument to justify the Rushbrooke's exponent identity: $\gamma + 2\beta = 2 - \alpha$. What is "hyperscaling"?

These questions have already been discussed in the lecture notes, and as part of the homework problems.

20 **Problem 4.** Assume that the conductivity σ at the metal-insulator transition obeys the following "quantum critical" scaling law:

$$\sigma(\delta n, T) = T^x f(T/\delta n^{\nu z}),$$

where $\delta n \sim (n - n_c)$ is the reduced density. Now consider the low temperature limit $T \to 0$, where the conductivity displays the critical behavior of the form:

$$\sigma(\delta n, 0) \sim \delta n^{\mu}.$$

Use a scaling argument to express the conductivity exponent μ, in terms of the exponents x and νz.

These questions have already been discussed in the lecture notes, and as part of the homework problems.