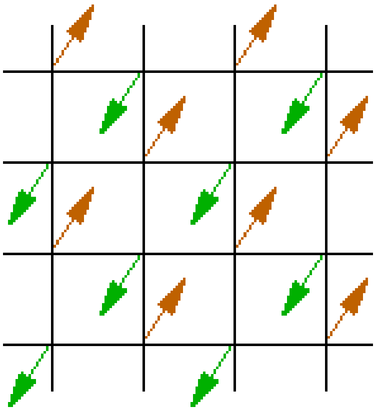
Other examples of critical behavior and symmetry breaking

Many different physical systems undergo critical phenomena and spontaneous symmetry breaking. Here, we just list some of the more important examples and comment how different phases can be identified by appropriate order parameters. We also mention examples of critical phenomena that show all features of second order phase transitions, yet no obvious static symmetry breaking pattern can easily be identified. Some of the examples include the glass transition and the disorder-driven metal-insulator transition. It appears that these new class of dynamical critical phenomena is emerging which are currently not well understood, but which will require radically new concepts and methods for their description.

Most physical systems showing spontaneous symmetry breaking of a static order parameter display critical behavior very similar to that of a ferromagnet. Some examples include :

- **Antiferromagnets.** In most insulating magnets (Mott insulators) there is a "super-exchange" interaction between spins that induces antiferromagnetic ordering - neighboring spins tend to anti allign. Typically, there are two sub-lattices, call them A and B, such that spins on one sub-lattice point opposite to that of the other sub-lattice, i.e. $m_A = -m_B$. The order parameter is the **staggered magnetization**, defined as the difference between the two

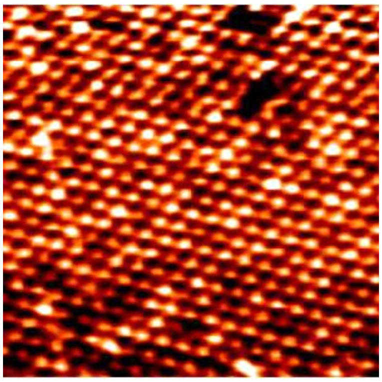


$$m^{\dagger} = m_A - m_B.$$

Such complicated ordering is not straightforward to identify by thermodynamic measurements, and in early days even the existence of antiferromagnetism was hotly debated. Even the famous Lev Landau did not believe in it for the longest time! In a typical antiferromagnet, the uniform spin susceptibility does not diverge anywhere, but instead has a cusp at the ordering ("Neel") temperature. The specific heat does diverge similarly as in ferromagnets. But the ultimate proof on antiferromagnetic ordering cannot be obtained for a given material before one identify the microscopic pattern of spins as they order at low temperature. In modern times this was made possible by using neutron scattering , muon spin relaxation, NMR, and other microscopic probes. Using these methods, one can directly measure the momentum -resolved spin susceptibility $\chi(\mathbf{q})$, which displays critical behavior, but only at the **ordering** wavevector $\mathbf{q} = \mathbf{Q}$, which describes the pattern ("wave") of spin ordering. recent years have seen an intense activity in the study of antiferromagnets, partly because most parent compounds from which one obtains (by doping) high temperature superconductors are antiferromagnetic (Mott) insulators such as $LaCuO_4$. It is believed that antiferromagnetic fluctuations play an important role in promoting superconducting pairing in these materials.

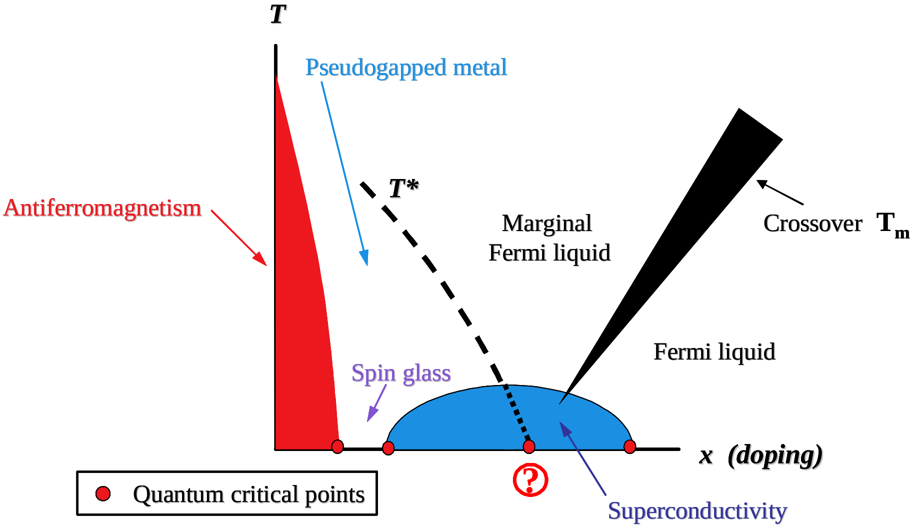
Landau did not believe in antiferromagnets

- **Charge density waves.** In crystals atoms (nuclei) assume a periodic pattern. However, in metals the conduction electrons are free to move throughout the material. In some cases, electron-electron interactions induce the electronic density to



assume a periodic modulation below some critical temperature. These electrons form a **charge density wave**. These often form on metal surfaces, and can be identified using, for example, scanning tunneling microscopy (STM). The STM pictures reveal the pattern the electrons assume. Such results have started to emerge in the last couple of years for cuprate superconductors (work of Seamus Davis and followers), which has re-ignited much debate about the possible role of charge ordering in cuprates.

- **Superconductors.** In superconductors the electrons pair up to form Cooper pairs, which Bose-condense and flow without resistance at low temperature. The phase transition is easy to identify - the resistivity simply drops to zero below T_c! The

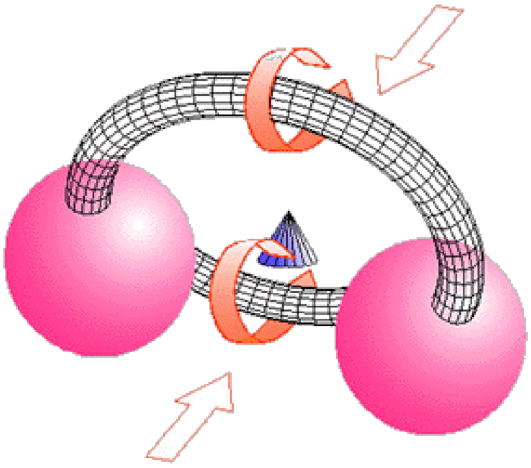


Phase diagram of high Tc superconductors

phenomenon was discovered at the beginning of the 20-th century but was not theoretically explained until the revolutionary work of Bardeen, Cooper and Schrieffer, who wrote down the famous BCS wavefunction in 1957. The connection of the BCS theory to the earlier phenomenological Landau-Ginzburg theory of superconductivity was established by Lev Gorkov in 1958. Both Schrieffer and Gorkov work at FSU! In superconductors, the order parameter is a complex phase describing the wavefunction of the condensate. As a result, thermal phase transitions in superconductors belong to the classical **XY model universality class**. There are many elemental metals and more complicated compounds that display superconductivity at low temperatures, and in most materials the BCS theory provides the complete theoretical description of the phenomenon. However, in most materials superconductivity emerges only at very low temperatures of a few degrees kelvin. More recently, a new class of high temperature superconductors have been found in 1987, which show superconductivity even above 100K! What precisely goes on in these materials is one of the biggest remaining mysteries of modern physics.

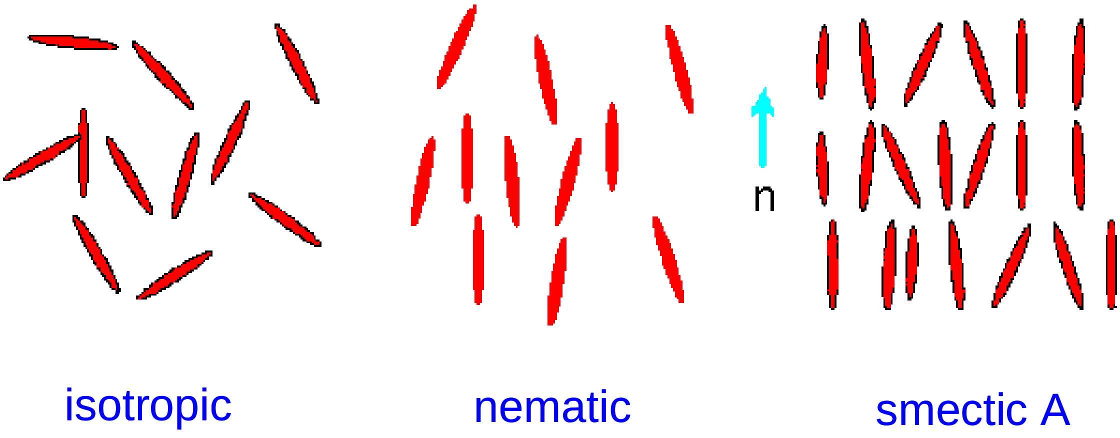
Interestingly, phenomena almost identical to superconductivity take place also within atomic nuclei. In fact, the ground state of many **nuclei** appears to display **pairing correlations**, and many features of their excitation spectra can be understood in terms of this physical picture.

- **Superfluids.** These include different low temperature phases of He^3 and He^4. Again, these belong to the XY model class.

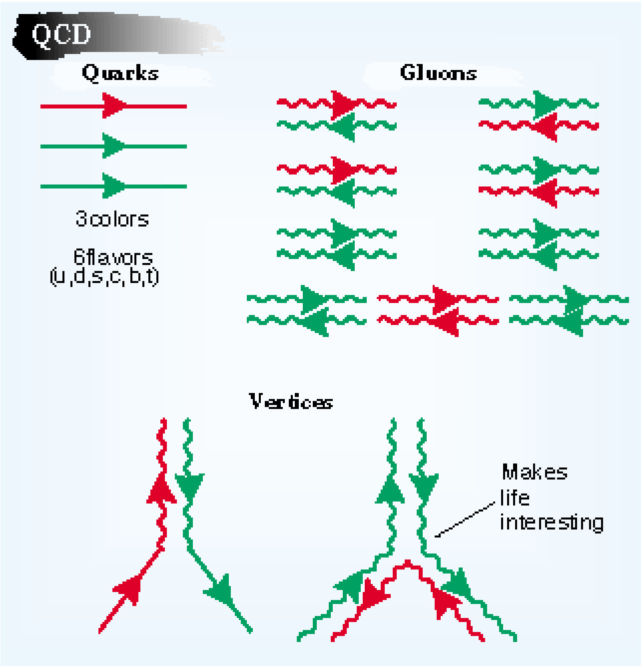


Vortex in superfluid He^4

- **Liquid crystals.** Here the order parameter is more complicated, corresponding to different ways how long rod-like molecules can orient themselves in a liquid. There are several different possibilities for ordering, including "nematic", "smectic", and other more complicated patterns.



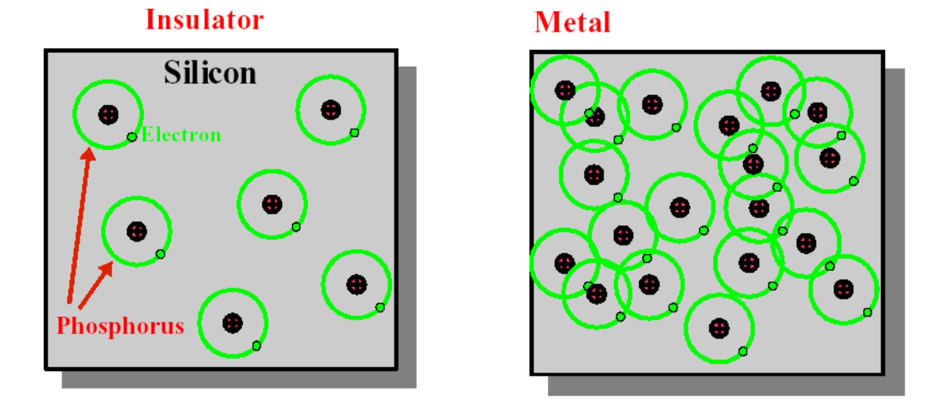
- **Quark deconfinement in cores of neutron stars.** Phase transitions involving spontaneous symmetry breaking are not limited to condensed matter or even nuclear physics. In fact, it is believed that elementary particles that form our World can be found in several phases, which diplay completely different types of matter. For

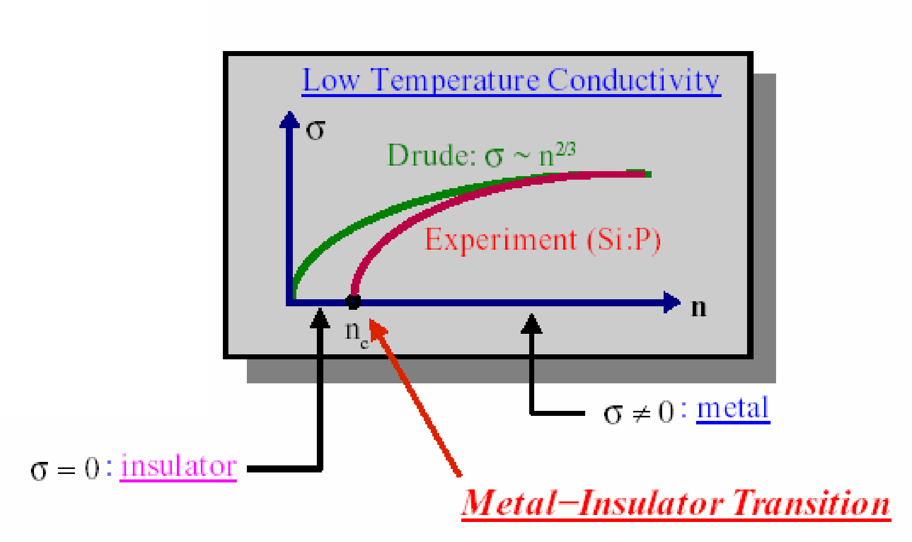


example, it is believed that nucleons such as protons and neutrons really are made up of more elementary **quarks**. Under current conditions, these quarks are "**confined",** i.e. the nucleons cannot be broken appart. However, it is also believed that other phases of matter may exists at much higher temperatures and pressures, where it is speculated that a **quark-gluon plazma** may exist. Recent work has suggested that such thermodynamic conditions may exist within the cores of neutrons stars, and much current research activity tries to prove or disprove this possibility. The theory that describes the behavior of quarks is called quantum chromo dynamics (QCD), and its predictions are currently being worked out to predice what different phases of matter may exist, and what their properties may be.

- **Spin, charge, and structural glasses.** In glasses the local degrees of freedom freeze in random directions. In contrast to uniform ordering, there is an exponentially large number of such low-energy configurations. Instead a few, the free energy develops a huge number of minima separated by a hierarchy of barriers. This leads to slow relaxation, inability to reach equilibrium over accessible time scales, and "aging" (memory of initial conditions even after a long time). In a glassy state, the system is trapped in one of many similar free energy minima, displaying an unusually complex pattern of ergodicity breaking (these are called "complex systems"). The mathematical formalism that attempts to describe such complexity is called "replica symmetry breaking".

- **Disorder-driven metal-insulator transitions.** Most materials conduct at finite temperature, simply because electrons can often overcome any kind of potential barier by thermal excitation. As the temperater is lowered the distinction between metals and insulators becomes more apparent. In metals the conductivity σ remains **finite** as $T \longrightarrow 0$. In insulators it drops to zero. A sharp phase transition between a metal and an insulator exists only at $T = 0$. This is our first encounter with **quantum phase transitions**. In materials such as the doped semiconductor Si:P (which is the basis for the semiconductor industry), one can go from an insulator to as metal as the carrier concentration is increased. Because the electrons are fermions, they obey the Pauli exclusion principle, and thus retain finite kinetic energy (comparable to their Fermi energy E_F) even at $T = 0$. As the carrier concentration increases, so





does their Fermi energy. At some point the corresponding kinetic energy of electrons is sufficient to overcome electron-electron or electron-impurity interactions, and electrons start to flow through the system. The behavior around the critical concentration is very similar to conventional criticla phenomena. For example the low temperature condcutivity behaves as

$$\sigma \sim (n - n_c)^{\mu},$$

where $n_c \approx 10^{18}$ cm^{-3} for Si:P, and the conductivity exponent $\mu \approx 1/2$. At present we do not have a consistent theory for this critical behavior, and no obvious static order parameter exists, in contrast to other critical phenomena.