ELECTRICITY AND MAGNETISM I

Homework set #9: Magnetostatics II

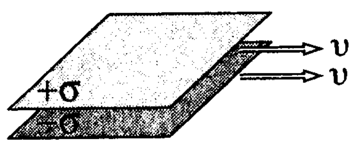
Problem # 9.1 :

A large parallel-plate capacitor with uniform surface charge σ on the upper plate and $-\sigma$ on the lower is moving with a constant speed v, as shown in the figure.

(a) Find the magnetic field between the plates and also above and below them.

(b) Find the magnetic force per unit area on the upper plate, including its direction.

(c) At what speed v would the magnetic force balance the electric force?



Problem # 9.2 :

(a) Find the density ρ of mobile charges in a piece of copper, assuming each atom contributes one electron (Cu forms a face centered cubic (fcc) lattice with the lattice parameter 3.61 Å).

(b) Calculate the average electron velocity in a copper wire 1 mm in diameter, carrying a current of 1 A.

(c) What is the force of attraction between two such wires, 1 cm apart?

(d) If you could somehow remove the stationary positive charges, what would the electrical repulsion force be? How many times greater than the magnetic force is it?

Problem # 9.3 :

Find the magnetic vector potential of a finite segment of straight wire carrying a current I. Put the wire along the z-axis, from z_1 to z_2, and use

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0 I}{4\pi} \int \frac{1}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{l}' \ .$$

Obtain the magnetic field $\mathbf{B}$ and check your answer with the result obtained in class:

$$\mathbf{B} = \frac{\mu_0 I}{4\pi s} [\sin(\theta_2) - \sin(\theta_1)] \hat{\phi} \ ,$$

where θ is the angle between $\mathbf{r} - \mathbf{r}'$ and the shortest distance from $\mathbf{r}$ to the line of the wire.

Problem # 9.4 :

(a) If $\mathbf{B}$ is *uniform*, show that $\mathbf{A}(\mathbf{r}) = -\frac{1}{2}(\mathbf{r} \times \mathbf{B})$ works. In other words, check that $\vec{\nabla} \cdot \mathbf{A} = 0$ and $\vec{\nabla} \times \mathbf{A} = \mathbf{B}$. Is this result unique, or are there other functions with the same divergence and curl? Give an example.

(b) As shown in class, the magnetic field of an infinite uniform surface current $\mathbf{K} = K\hat{\mathbf{x}}$, flowing over the xy plane, is given by

$$\mathbf{B} = -\frac{\mu_0}{2} K \text{sign}(z) \hat{\mathbf{y}} \ ,$$

where *sign* is the sign-function. Find the vector potential above and below the plane surface current.

Problem # 9.5 :

(a) Show that the magnetic field of a dipole can be written in coordinate free form:

$$\mathbf{B}_{dip}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{1}{r^3} \left[3(\mathbf{m} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m} \right] \ .$$

(b) A circular loop of wire, with radius R, lies in the xy plane (centered at the origin) and carries a current I running counterclockwise as viewed from the positive z axis.

(1) What is the dipole moment?

(2) What is the approximate magnetic field at points far from the origin?

(3) Show that, for points on the z axis, your answer is consistent with the *exact* field obtained in class

$$B(z) = \frac{\mu_0 I}{2} \frac{R^2}{(R^2 + z^2)^{3/2}} \ ,$$

when $z \gg R$.