

**Physics 5645**  
**Quantum Mechanics A**  
**Problem Set II**

Due: Tuesday, Sep 26, 2017

2.1 Problem 1.4, Sakurai and Napolitano, Pg. 59.

2.2 Problem 1.11, Sakurai and Napolitano, Pg. 60.

2.3 Problem 1.12, Sakurai and Napolitano, Pg. 60.

2.4 As an example of a system described by a three-dimensional Hilbert space consider the case of a spin-1 particle. For such a particle the matrix representations of  $S_x$ ,  $S_y$ , and  $S_z$  in the  $S_z$  basis are,

$$S_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_y = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad S_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

(a) Obtain the eigenvalues and corresponding eigenvectors of  $S_x$  and  $S_y$  in the  $S_z$  basis.

(b) Consider a system prepared in the state

$$|\psi\rangle = A(|+\rangle + i|0\rangle + |-\rangle)$$

where  $S_z|+\rangle = \hbar|+\rangle$ ,  $S_z|0\rangle = 0|0\rangle$ ,  $S_z|-\rangle = -\hbar|-\rangle$ , and  $A$  is a normalization constant.

Determine the probabilities for all possible measurement results of  $S_x$  for this state.

(c) Assume that the result of the measurement made in (b) is  $S_x = +\hbar$ , and that immediately after this measurement  $S_y$  is measured. Determine the probabilities for all possible results of this  $S_y$  measurement.

(d) Now assume the system is prepared in a state with  $S_z = +\hbar$ . Determine the expectation values  $\langle S_x \rangle$ ,  $\langle S_y \rangle$ ,  $\langle S_x^2 \rangle$ , and  $\langle S_y^2 \rangle$  for this state. Obtain the uncertainties  $\langle (\Delta S_x)^2 \rangle = \langle S_x^2 \rangle - \langle S_x \rangle^2$  and  $\langle (\Delta S_y)^2 \rangle = \langle S_y^2 \rangle - \langle S_y \rangle^2$  and verify that your result satisfies the generalized uncertainty relation.

2.5 Problem 1.19, Sakurai and Napolitano, Pg. 61.

2.6 Problem 1.26, Sakurai and Napolitano, Pg. 63.