# Physics 5645
# Quantum Mechanics A
# Problem Set III

Due: Tuesday, Oct 3, 2017

3.1 As I'm sure you know, and we will soon see, the eigenkets of the Hamiltonian operator for a one-dimensional particle confined between two rigid walls, one at $x = 0$ and the other at $x = a$, are $|\psi_n\rangle = \int \psi_n(x)|x\rangle dx$ where

$$\psi_n(x) = \begin{cases} A \sin \frac{n\pi x}{a} & \text{for } 0 < x < a, \\ 0 & \text{otherwise.} \end{cases}$$

Here $n = 1, 2, 3, \cdots$ and $A$ is a normalization constant.

Compute the $x$-$p$ uncertainty products $\langle(\Delta x)^2\rangle\langle(\Delta p)^2\rangle$ for the states $|\psi_n\rangle$ and verify that they satisfies the uncertainty principle for all $n$.

3.2 Show that if the position-space wave function for a particle, $\psi(x) = \langle x|\psi\rangle$, is real valued then the expectation value of the momentum operator in that state is $\langle\hat{p}\rangle = 0$. (Hint: Show that the probabilities to measure momenta of $+p$ and $-p$ are equal.) Also show that multiplying $\psi(x)$ by a constant, even if complex, does not change this result. (It better not, since the states $|\psi\rangle$ and $c|\psi\rangle$ are physically equivalent.)

3.3 Problem 1.27, Sakurai and Napolitano, Pg. 63.

3.4 Problem 1.32, Sakurai and Napolitano, Pg. 65.

3.5 A one-dimensional particle is in the state $|\psi\rangle = \int \psi(x)|x\rangle dx$ where $\psi(x) = \langle x|\psi\rangle$, the position-space wave function, is given by

$$\psi(x) = \begin{cases} A & -a \leq x \leq a, \\ 0 & \text{otherwise.} \end{cases}$$

**(a)** Find $A$ so that $|\psi\rangle$ is normalized and compute the momentum-space wave function for this state, $\tilde{\psi}(p) = \langle p|\psi\rangle$.

**(b)** If an experiment were performed to measure the momentum $p$ of the particle in this state, what is the probability that the result would be such that $|p| \geq \hbar/a$? [To answer this you may need to do an integral numerically.]