2.4 As an example of a system described by a three-dimensional Hilbert space consider the case of a spin-1 particle. For such a particle the matrix representations of $S_x$, $S_y$, and $S_z$ in the $S_z$ basis are,

$$S_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_y = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad S_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

(a) Obtain the eigenvalues and corresponding eigenvectors of $S_x$ and $S_y$ in the $S_z$ basis.

(b) Consider a system prepared in the state

$$|\psi\rangle = A \left( |+\rangle + i|0\rangle + |-\rangle \right)$$

where $S_z|+\rangle = \hbar|+\rangle$, $S_z|0\rangle = 0|0\rangle$, $S_z|-\rangle = -\hbar|-\rangle$, and $A$ is a normalization constant. Determine the probabilities for all possible measurement results of $S_x$ for this state.

(c) Assume that the result of the measurement made in (b) is $S_x = +\hbar$, and that immediately after this measurement $S_y$ is measured. Determine the probabilities for all possible results of this $S_y$ measurement.

(d) Now assume the system is prepared in a state with $S_z = +\hbar$. Determine the expectation values $\langle S_x \rangle$, $\langle S_y \rangle$, $\langle S^2_x \rangle$, and $\langle S^2_y \rangle$ for this state. Obtain the uncertainties $\langle (\Delta S_x)^2 \rangle = \langle S^2_x \rangle - \langle S_x \rangle^2$ and $\langle (\Delta S_y)^2 \rangle = \langle S^2_y \rangle - \langle S_y \rangle^2$ and verify that your result satisfies the generalized uncertainty relation.

2.5 Problem 1.19, Sakurai and Napolitano, Pg. 61.

2.6 Problem 1.26, Sakurai and Napolitano, Pg. 63.