Physics 5645
Quantum Mechanics A
Problem Set III

Due: Tuesday, Sep 25, 2018, DEADLINE EXTENDED TO **TUESDAY, OCT 2.**

3.1 Problem 1.4, Parts (c) and (d), Sakurai and Napolitano, Pg. 59.

3.2 In Problem 1.3 (Problem 1.8 in Sakurai and Napolitano) you showed that

$$[S_i, S_j] = i\epsilon_{ijk}\hbar S_k \quad \text{and} \quad \{S_i, S_j\} = \left(\frac{\hbar^2}{2}\right)\delta_{ij},$$

or, since $\vec{S} = \frac{\hbar}{2}\vec{\sigma}$,

$$[\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma_k \quad \text{and} \quad \{\sigma_i, \sigma_j\} = 2\delta_{ij}\mathbb{1},$$

where σ_i are the Pauli matrices [the index i runs over Cartesian components x, y, and z and repeated indices are summed] and $\mathbb{1}$ is the 2×2 identity matrix.

(a) Show that

$$\sigma_i\sigma_j = \delta_{ij}\mathbb{1} + i\epsilon_{ijk}\sigma_k. \tag{1}$$

(b) Show that (1) together with the facts that $Tr[\mathbb{1}] = 2$ and $Tr[\sigma_i] = 0$ implies

$$Tr[\sigma_i\sigma_j] = 2\delta_{ij}.$$

Now consider a general 2×2 matrix (not necessarily Hermitian or Unitary) of the form

$$A = a_0\mathbb{1} + \vec{a}\cdot\vec{\sigma}$$

where a_0 and a_i $(i = x, y, z)$ are complex numbers.

(c) How are a_0 and a_i related to $Tr[A]$ and $Tr[\sigma_i A]$?

(d) Obtain a_0 and a_i in terms of the matrix elements of $A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$.

[**Note:** This problem gives a useful method for showing things like

$$\begin{pmatrix} H_{11} & H_{12} \\ H_{12} & H_{22} \end{pmatrix} = \frac{H_{11}+H_{22}}{2}\mathbb{1} + \frac{H_{11}-H_{22}}{2}\sigma_z + H_{12}\sigma_x,$$

which was useful for doing Problem 1.11 from Sakurai and Napolitano on the last Problem Set.]

3.3 Problem 1.16, Sakurai and Napolitano, Pg. 61.

3.4 As I'm sure you know, and we will soon see, the eigenkets of the Hamiltonian operator for a one-dimensional particle confined between two rigid walls, one at $x = 0$ and the other at $x = a$, are $|\psi_n\rangle = \int \psi_n(x)|x\rangle dx$ where

$$\psi_n(x) = \begin{cases} A \sin \frac{n\pi x}{a} & \text{for } 0 < x < a, \\ 0 & \text{otherwise.} \end{cases}$$

Here $n = 1, 2, 3, \cdots$ and A is a normalization constant.

Compute the x-p uncertainty products $\langle(\Delta x)^2\rangle\langle(\Delta p)^2\rangle$ for the states $|\psi_n\rangle$ and verify that they satisfies the uncertainty principle for all n.

3.5 Problem 1.32 Part (a), Sakurai and Napolitano, Pg. 61. [You will do Part (b) of this problem on the next Problem Set.]