Physics 5645
Quantum Mechanics A
Problem Set IV

Due: Tuesday, Oct 9, 2018

4.1 Show that if the position-space wave function for a particle, $\psi(x) = \langle x|\psi\rangle$, is real valued then the expectation value of the momentum operator in that state is $\langle \hat{p}\rangle = 0$. (Hint: Show that the probabilities to measure momenta of $+p$ and $-p$ are equal.) Also show that multiplying $\psi(x)$ by a constant, even if complex, does not change this result. (It better not, since the states $|\psi\rangle$ and $c|\psi\rangle$ are physically equivalent.)

4.2 Problem 1.27, Sakurai and Napolitano, Pg. 63.

4.3 Problem 1.32 Part (b), Sakurai and Napolitano, Pg. 65.

4.4 A one-dimensional particle is in the state $|\psi\rangle = \int \psi(x)|x\rangle dx$ where $\psi(x) = \langle x|\psi\rangle$, the position-space wave function, is given by

$$\psi(x) = \begin{cases} A & -a \le x \le a, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find A so that $|\psi\rangle$ is normalized and compute the momentum-space wave function for this state, $\tilde{\psi}(p) = \langle p|\psi\rangle$.

(b) If an experiment were performed to measure the momentum p of the particle in this state, what is the probability that the result would be such that $|p| \ge \hbar/a$? [To answer this you may need to do an integral numerically.]

4.5 Problem 2.2, Sakurai and Napolitano, Pg. 148.

4.6 Consider the precession of the spin of an electron in a uniform magnetic field $\vec{B} = B\hat{k}$ for which the Hamiltonian is

$$H = -\frac{eB}{mc}S_z = \omega S_z,$$

where $\omega = \frac{|e|B}{mc}$.

At time $t = 0$ the electron spin is in the state,

$$|\psi(0)\rangle = \cos\frac{\theta}{2}|+\rangle + \sin\frac{\theta}{2}|-\rangle,$$

(i.e., an eigenstate of the operator $\hat{\mathbf{n}}\cdot\mathbf{S}$ with eigenvalue $+\hbar/2$ where $\hat{\mathbf{n}}$ is in the xz plane and makes an angle θ with the z axis.) In what follows work within the Schrödinger picture.

(a) Obtain the time dependent state $|\psi(t)\rangle$.

(b) Find the probability for finding the electron in the $S_x = \hbar/2$ state as a function of time.

(c) Obtain the expectation values of S_x, S_y, and S_z as a function of time. Describe the time evolution of the vector $\langle\mathbf{S}\rangle = \langle\psi(t)|\mathbf{S}|\psi(t)\rangle$.

4.7 Time Evolution of free particle wave packet (Schrödinger picture).

Consider the motion of a free particle in one-dimension with Hamiltonian

$$H = \frac{\hat{p}^2}{2m}.$$

At time $t = 0$ the particle is in a state $|\psi(0)\rangle$ with position-space wave function

$$\langle x|\psi(0)\rangle = \frac{1}{\pi^{1/4}\sqrt{d}}\exp^{-x^2/(2d^2)}\, e^{ikx}.$$

(a) Working in the Schrödinger picture, find the position space wave function at time t.

Answer:

$$\langle x|\psi(t)\rangle = \frac{1}{\pi^{1/4}\sqrt{d}}\frac{\exp^{-(x-v_g t)^2/\left[2d^2\left(1+i\frac{\hbar}{md^2}t\right)\right]}}{\sqrt{1+i\frac{\hbar}{md^2}t}}e^{ik(x-v_p t)}, \tag{1}$$

where $v_g = \hbar k/m$ is the group velocity, and $v_p = \hbar k/(2m)$ is the phase velocity.

(b) Specializing to the case $k = 0$, compute the expectation values $\langle\hat{x}\rangle$, $\langle\hat{x}^2\rangle$, $\langle\hat{p}\rangle$, and $\langle\hat{p}^2\rangle$. Determine $\langle(\Delta x)^2\rangle$ and $\langle(\Delta p)^2\rangle$ as a function of time and verify that the uncertainty principle always holds.