Physics 5645
Quantum Mechanics A
Problem Set VI

Due: Tuesday, Oct 30, 2018

6.1 Obtain the energy levels and normalized position-space wave functions for the energy eigenstates of an infinite square well of width L centered at $x = L/2$. Imagine the particle is in the state $\psi(x) = Ax(x - L)$ at time $t = 0$. If the energy of the particle is then measured, obtain an expression for the probability to find the system in any of the energy eigenstates.

6.2 Consider a particle in the finite square well potential

$$V(x) = \begin{cases} 0, & -a \le x \le a \\ V_0, & \text{otherwise} \end{cases}$$

where $V_0 > 0$.

Because $V(x) = V(-x)$ we know that the bound state solutions will have either even or odd parity and so must have the forms,

	Even Parity	Odd Parity
$x \le -a$	$\psi(x) = Ae^{\rho x}$	$\psi(x) = Ae^{\rho x}$
$-a < x < a$	$\psi(x) = B\cos(kx)$	$\psi(x) = B\sin(kx)$
$x \ge a$	$\psi(x) = Ae^{-\rho x}$	$\psi(x) = -Ae^{-\rho x}$

where $k = \sqrt{2mE/\hbar^2}$ and $\rho = \sqrt{2m(V_0 - E)/\hbar^2}$.

In class we showed that even parity bound states can be found by solving the equation

$$\tan z = +\sqrt{\left(\frac{z_0}{z}\right)^2 - 1},$$

where $z = ka$ and $z_0 = \sqrt{\frac{2mV_0}{\hbar^2}}a$.

(a) Show that the odd parity bound states can be found by solving the equation

$$-\cot z = +\sqrt{\left(\frac{z_0}{z}\right)^2 - 1}.$$

Now consider the *half*-finite square well with potential

$$V(x) = \begin{cases} \infty, & x < 0, \\ 0, & 0 \le x \le a, \\ V_0, & x > a. \end{cases}$$

(b) How are the bound states of this potential related to those of the finite square well of Part (a)?

(c) Are there bound states of this potential for any $V_0 > 0$? If not, what is the maximum value of V_0 for which there are no bound states?

6.3 Probability current.

(a) Show that the probability current density for a one-dimensional quantum particle with position-space wave function

$$\psi(x) = Ae^{ikx} + Be^{-ikx},$$

where k is real valued, is

$$j = \frac{\hbar k}{m}\left(|A|^2 - |B|^2\right).$$

(b) Show that the probability current density for any particle with a real-valued position space wave function is zero.

6.4 Delta function potential.

Consider a one-dimensional quantum particle of mass m in the presence of a delta-function potential $V(x) = -\alpha\delta(x)$ where α has units of energy times length.

(a) By applying the appropriate boundary conditions at $x = 0$, show that for $\alpha > 0$ and $E < 0$ this potential admits a bound state of energy $E = -m\alpha^2/(2\hbar^2)$. Are there any other bound states?

(b) Now consider a scattering process. Seek solutions for $E > 0$ of the form $\psi(x) = Ae^{ikx} + Be^{-ikx}$ for $x < 0$ and $\psi(x) = Ce^{ikx}$ for $x > 0$. By applying the appropriate boundary conditions at $x = 0$, obtain the reflection and transmission coefficients R and T for this potential.

6.5 Consider a one-dimensional quantum particle in the ground state of an infinite square well of width L. Suddenly, the well is expanded symmetrically to twice its size, leaving the wave function undisturbed. What is the probability to find the particle in the ground state of the new well?