Physics 5645
Quantum Mechanics A
Problem Set II

Due: Tuesday, Sep 17, 2019

2.1 Problem 1.4, Sakurai and Napolitano, Pg. 59.

2.2 Problem 1.11, Sakurai and Napolitano, Pg. 60.

2.3 Problem 1.12, Sakurai and Napolitano, Pg. 60.

2.4 As an example of a system described described by a three-dimensional Hilbert space consider the case of a spin-1 particle. For such a particle the matrix representations of S_x, S_y, and S_z in the S_z basis are,

$$S_x = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad S_y = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad S_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

(a) Obtain the eigenvalues and corresponding eigenvectors of S_x and S_y in the S_z basis.

(b) Consider a system prepared in the state

$$|\psi\rangle = A\left(|+\rangle + i|0\rangle + |-\rangle\right)$$

where $S_z|+\rangle = \hbar|+\rangle$, $S_z|0\rangle = 0|0\rangle$, $S_z|-\rangle = -\hbar|-\rangle$, and A is a normalization constant. Determine the probabilities for all possible measurement results of S_x for this state.

(c) Assume that the result of the measurement made in (b) is $S_x = +\hbar$, and that immediately after this measurement S_y is measured. Determine the probabilities for all possible results of this S_y measurement.

(d) Now assume the system is prepared in a state with $S_z = +\hbar$. Determine the expectation values $\langle S_x\rangle$, $\langle S_y\rangle$, $\langle S_x^2\rangle$, and $\langle S_y^2\rangle$ for this state. Obtain the uncertainties $\langle(\Delta S_x)^2\rangle = \langle S_x^2\rangle - \langle S_x\rangle^2$ and $\langle(\Delta S_y)^2\rangle = \langle S_y^2\rangle - \langle S_y\rangle^2$ and verify that your result satisfies the generalized uncertainty relation.

2.5 Problem 1.19, Sakurai and Napolitano, Pg. 61.

2.6 Problem 1.26, Sakurai and Napolitano, Pg. 63.