

Physics 5645
Quantum Mechanics A
Problem Set VI

Due: Tuesday, Oct 22, 2019

6.1 Obtain the energy levels and normalized position-space wave functions for the energy eigenstates of an infinite square well of width L centered at $x = L/2$. Imagine the particle is in the state $\psi(x) = Ax(x - L)$ at time $t = 0$. If the energy of the particle is then measured, obtain an expression for the probability to find the system in any of the energy eigenstates.

6.2 Consider a particle in the finite square well potential

$$V(x) = \begin{cases} 0, & -a \leq x \leq a \\ V_0, & \text{otherwise} \end{cases}$$

where $V_0 > 0$.

Because $V(x) = V(-x)$ we know that the bound state solutions will have either even or odd parity and so must have the forms,

	Even Parity	Odd Parity
$x \leq -a$	$\psi(x) = Ae^{\rho x}$	$\psi(x) = Ae^{\rho x}$
$-a < x < a$	$\psi(x) = B \cos(kx)$	$\psi(x) = B \sin(kx)$
$x \geq a$	$\psi(x) = Ae^{-\rho x}$	$\psi(x) = -Ae^{-\rho x}$

where $k = \sqrt{2mE/\hbar^2}$ and $\rho = \sqrt{2m(V_0 - E)/\hbar^2}$.

In class we showed that even parity bound states can be found by solving the equation

$$\tan z = +\sqrt{\left(\frac{z_0}{z}\right)^2 - 1},$$

where $z = ka$ and $z_0 = \sqrt{\frac{2mV_0}{\hbar^2}}a$.

(a) Show that the odd parity bound states can be found by solving the equation

$$-\cot z = +\sqrt{\left(\frac{z_0}{z}\right)^2 - 1}.$$

Now consider the *half*-finite square well with potential

$$V(x) = \begin{cases} \infty, & x < 0, \\ 0, & 0 \leq x \leq a, \\ V_0, & x > a. \end{cases}$$

(b) How are the bound states of this potential related to those of the finite square well of Part (a)?

(c) Are there bound states of this potential for any $V_0 > 0$? If not, what is the maximum value of V_0 for which there are no bound states?

6.3 Probability current.

(a) Show that the probability current density for a one-dimensional quantum particle with position-space wave function

$$\psi(x) = Ae^{ikx} + Be^{-ikx},$$

where k is real valued, is

$$j = \frac{\hbar k}{m} (|A|^2 - |B|^2).$$

(b) Show that the probability current density for any particle with a real-valued position space wave function is zero.

6.4 Delta function potential.

Consider a one-dimensional quantum particle of mass m in the presence of a delta-function potential $V(x) = -\alpha\delta(x)$ where α has units of energy times length.

(a) By applying the appropriate boundary conditions at $x = 0$, show that for $\alpha > 0$ and $E < 0$ this potential admits a bound state of energy $E = -m\alpha^2/(2\hbar^2)$. Are there any other bound states?

(b) Now consider a scattering process. Seek solutions for $E > 0$ of the form $\psi(x) = Ae^{ikx} + Be^{-ikx}$ for $x < 0$ and $\psi(x) = Ce^{ikx}$ for $x > 0$. By applying the appropriate boundary conditions at $x = 0$, obtain the reflection and transmission coefficients R and T for this potential.

6.5 Consider a one-dimensional quantum particle in the ground state of an infinite square well of width L . Suddenly, the well is expanded symmetrically to twice its size, leaving the wave function undisturbed. What is the probability to find the particle in the ground state of the new well?