Physics 5646
Quantum Mechanics B
Problem Set IV

Due: Tuesday, Feb 13, 2018

4.1 Consider the Hamiltonian,

$$H = H_0 + \lambda \hat{p}^2,$$

where

$$H_0 = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2,$$

is the Hamiltonian for a 1D harmonic oscillator.

(a) Treat the term proportional to λ as a perturbation and calculate the first- and second-order energy shifts of the nth energy level of this oscillator.

(b) Now consider the full Hamiltonian H as a harmonic oscillator with a modified mass and frequency. Determine the modified frequency and from that determine the exact energy levels of the system. Expand these energy levels to appropriate order in λ and compare your results to those found in Part (a).

4.2 Consider an anharmonic oscillator with Hamiltonian,

$$H = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2 \hat{x}^2 + \alpha \hat{x}^4.$$

Treating the quartic term as a perturbation compute the first-order energy shift of the nth energy level.

4.3 The Hamiltonian for a spin-1/2 particle with gyromagnetic ratio γ in a magnetic field $\mathbf{B} = B\hat{i} + B_0\hat{k}$ is

$$H = -\gamma \mathbf{S} \cdot \mathbf{B} = -\frac{\hbar\gamma}{2} (B_0 \sigma_z + B\sigma_x).$$

(a) Obtain the exact eigenvalues and normalized eigenvectors of H.

(b) Now, treating B as a perturbation and using non-degenerate perturbation theory, compute the eigenvalues of H up to second order, and the eigenvectors of H up to first order.

(c) Compare the exact eigenvalues and eigenvectors you obtained in Part (a), expanded to appropriate order, to the perturbation theory results you obtained in Part (b).

4.4 In solving the hydrogen atom problem we made the idealizing assumption that the proton is a point charge. To account for the fact that this is not the case, we can model the proton as a sphere of total charge $+e$ and radius R ($\simeq$ 1 fm) with uniform charge density.

The potential energy of the electron in a hydrogen atom is then

$$V(r) = \begin{cases} -\frac{3e^2}{2R} + \frac{e^2 r^2}{2R^3}, & r \leq R \\ -\frac{e^2}{r}, & r > R \end{cases}$$

Calculate the first-order shift to the ground state energy of hydrogen due to this modification. You may assume that $R << a_0$ so that $e^{-R/a_0} \simeq 1$.

4.5 Consider two identical spinless bosons of mass m in an infinite square well of length L with one-particle eigenstates $\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$, for $0 \leq x \leq L$, and one-particle energies $E_n = \frac{\hbar^2 \pi^2}{2mL^2} n^2$; n= 1,2,3,$\cdots$.

First assume the bosons are not interacting.

(a) Determine the position-space wave functions and energies of the ground state and first excited state of this system.

Now assume the bosons interact via a delta function interaction $V(x_1, x_2) = \alpha L \delta(x_1 - x_2)$ where α has dimensions of energy.

(b) Compute the first-order energy shifts for the ground state and first excited state of this system.