Physics 5646
Quantum Mechanics B
Problem Set III

Due: Tuesday, Feb 5, 2019

3.1 You are given the following H-atom matrix element,

$$\langle 210|\hat{p}_z|100\rangle = i\frac{16\sqrt{2}}{81}\frac{\hbar}{a_0}.$$

Here $\hat{\mathbf{p}}$ is the momentum operator, a_0 is the Bohr radius, and the standard H-atom eigenstates are labeled $|nlm\rangle$, where n, l, and m are the usual H-atom quantum numbers (ignoring spin).

(a) Using the Wigner-Eckart theorem, determine the matrix element $\langle 211|\hat{p}_y|100\rangle$.

(b) Compute the expectation value of $\hat{p}_y$ in the state $\frac{1}{\sqrt{2}}(|100\rangle + |211\rangle)$.

3.2 Consider the problem of adding angular momentum j_1 to j_1 for which the allowed values of the total angular momentum are $j = 2j_1, 2j_1 - 1, \cdots, 0$.

(a) Show the states with $j = 2j_1$ are symmetric under spin exchange.

(b) Show that the states with $j = 2j_1 - 1$ are antisymmetric under spin exchange.

Hint: In both cases, argue that the state with the highest m value has the given symmetry, and that applying J_- does not change this symmetry.

3.3 Consider two noninteracting particles of mass m in a harmonic potential well $V(x) = \frac{1}{2}m\omega^2x^2$. For the case with one particle in the state $|n\rangle$ and the other in the state $|l\rangle$ (where $|n\rangle$ and $|l\rangle$ are one-particle harmonic oscillator eigenstates, in the usual notation, and $n \neq l$), calculate the expectation value $\langle(\hat{x}_1 - \hat{x}_2)^2\rangle$ for the following cases:

(a) The particles are distinguishable.

(b) The particles are identical spin-0 bosons.

(c) The particles are identical spin-1/2 fermions in a singlet state.

(d) The particles are identical spin-1/2 fermions in a triplet state.

3.4 Consider two noninteracting spin-1 bosons in an infinite square well of length L for which the energy levels are $E_n = \frac{\hbar^2\pi^2}{2mL^2}n^2$, for $n = 1, 2, 3, \cdots$.

(a) What is the ground state energy of this system? What are the allowed values for the total spin in the ground state, and what is the degeneracy of this state?

(b) What is the energy of the first excited state of this system? Again, what are the allowed values for the total spin in this excited state, and what is the degeneracy of this state?

3.5 Consider a system in which there are three particles and only three states $|a\rangle$, $|b\rangle$, and $|c\rangle$, available to them. Show that the number of allowed, distinct configurations for this system is

(a) 27 if the particles are distinguishable,

(b) 10 if the particles are identical bosons,

(c) 1 if the particles are identical fermions.